

by

James L. Allen  
 Electrical and Electronic Systems  
 University of South Florida  
 Tampa, Florida 33620

Abstract

Experimental and Theoretical results are presented demonstrating some novel properties of inhomogeneous coupled-line structures having large even-odd mode velocity ratios, e.g., a single inhomogeneous coupled-line filter section with an equiripple, 3-peak stopband response.

Introduction

Coupled-line structures are utilized extensively as building blocks for filters, directional couplers and other important transmission line devices.<sup>1,2</sup> Odd and even propagation mode components<sup>3</sup> are commonly used in analyzing coupled lines. The phase velocities associated with the odd and even modes are equal for lines in a homogeneous medium but unequal for lines in an inhomogeneous medium. Coupled lines in micro-strip<sup>4</sup> and suspended-substrate stripline<sup>5</sup> yield small deviations from equal velocities (velocity ratios typically less than 1.2). Much larger deviations can be obtained with inhomogeneous broadside-coupled strips<sup>6</sup> (velocity ratios of 3.0 and even larger). Velocity ratios of roughly 2 to 1 and higher can lead to dramatic useful changes in the performance of familiar circuits.

The purpose of this paper is to present experimental and theoretical results demonstrating some of the novel properties of inhomogeneous coupled-line structures having large even-odd mode velocity ratios. For example, it will be shown that a rippled pass and stopband response can be realized with a single inhomogeneous section producing a response similar to that of several homogeneous sections in cascade.

Analysis

The inhomogeneous counterpart of each of the ten two-port coupled-line configurations identified by Jones and Bolljahn<sup>7</sup> has been studied. Each configuration exhibits interesting properties not possessed by the homogeneous version. For the purposes of this summary, attention will be focused on one configuration, the C-section as shown schematically in Figure 1A. In the homogeneous case of equal mode velocities a C-section is an all-pass network. As shown below, an inhomogeneous C-section with a large even-odd mode velocity ratio becomes a very good bandpass network. Information similar to that to be presented for the C-section is available for the other configurations.

Design relations for the inhomogeneous coupled-line filters were obtained by analytically manipulating the insertion loss expression as given in terms of the ABCD parameters of the network and the generator and load impedances. The ABCD parameters of a pair of inhomogeneous coupled lines can be determined by the method of Jones and Bolljahn<sup>7</sup>. Results for symmetrical lines are given by Zysman and Johnson<sup>8</sup>.

General equations have been derived for each type section. The features of these relationships are well illustrated by the specific case of the C-section

with a 2 to 1 velocity ratio.

Define:

$Z_{oe}$  = even mode impedance  
 $Z_{oo}$  = odd mode impedance  
 $v_e$  = even mode velocity  
 $v_o$  = odd mode velocity  
 $\ell$  = physical length of coupled section  
 $Z_o$  = characteristic impedance of input and output lines  
 $k = v_e/v_o, R = Z_{oe}/Z_{oo}, M = Z_o^2/Z_{oe}Z_{oo}$   
 $\theta_e = 2\pi f \ell / v_e$  = even mode electrical length  
 $\theta_o = 2\pi f \ell / v_o$  = odd mode electrical length =  $k\theta_e$

For the C-section with  $k=2$ , the locations of zeros of attenuation are given by

$$(1A) \theta_e = 0^\circ, 180^\circ, 360^\circ, \dots \text{ and}$$

$$(1B) \theta_e = \sin^{-1} \left( \pm \sqrt{\frac{M-2}{2M-2}} \right) \text{ for } M \geq 2.$$

If  $M < 2$  only the  $\theta_e = 0^\circ, 180^\circ, 360^\circ, \dots$  zeros are present. Locations of poles of attenuation are given by

$$(2A) \theta_e = 90^\circ, 270^\circ, \dots \text{ and}$$

$$(2B) \theta_e = \cos^{-1} \left( \pm \sqrt{\frac{R-2}{2R-2}} \right) \text{ for } R \geq 2.$$

For  $R < 2$  only the  $\theta_e = 90^\circ, 270^\circ, \dots$  poles are present. Relative maxima and minima are located where

$$(3) \theta_e = \sin^{-1} \left( \pm \sqrt{x} \right)$$

$$\text{where } x = (-B \pm \sqrt{B^2 - 4AC})/(2A)$$

$$\text{and } A = 4(RM-1)$$

$$B = 2(R-M) + 4(1-RM)$$

$$C = R(M-2).$$

Only positive values of  $x \leq 1$  are admissible.

The value of attenuation at each relative maximum or minimum is given by

$$(4) (\text{Insertion Loss})_{db} = 10 \log_{10} \left[ 1 + \frac{R}{M} \left( \frac{C_1 C_2}{C_3} \right)^2 \right]$$

$$\text{where } C_1 = \frac{x}{1-x}, C_2 = 2x - 2Mx + M-2,$$

$$C_3 = 2x - 2xR + R$$

and  $x$  is given in Equation 3.

Graphs of the design equations for the C-section with  $k = 2$  are given in Figures 2 and 3. The following comments pertain to these graphs. Notice that zeros are always located at  $\theta_e = 0^\circ, 180^\circ, \dots$ . For  $M > 2$  an additional pair of zeros is present in each  $180^\circ$  period of  $\theta_e$  yielding a wider passband with ripples. For example, if  $M = 3$ , zeros are located at  $\theta_e = 0^\circ, 30^\circ, 150^\circ, \dots$ , etc.

Similarly a pole of attenuation is always located at  $\theta = 90^\circ$ . For  $R > 2$  an additional pair of poles is present in each  $180^\circ$  period of  $\theta_e$  yielding a wider stopband with ripples. For example, if  $R = 3$  poles

are located at  $\theta_e = 60^\circ, 90^\circ, 120^\circ$ .

Finite, nonzero relative maxima and/or minima of attenuation are present if  $M \geq 2, R \geq 2$ . For example with  $M = 3, R = 3$  relative minima are located in the stopband at  $\theta_e = 65.2^\circ, 114.8^\circ$  while relative maxima are located in the passband at  $\theta_e = 5.3^\circ, 174.7^\circ$ . The peak value of attenuation in the passband is the value at each maxima or 0.025 db. The minimum value of attenuation in the stopband is the value at each minima or 22.4 db.

The overall calculated response of an inhomogeneous C-section with  $k = 2, M = 3, R = 3, Z_0 = 50, l = 1.875$  cm,  $v_0 = 1.14 \times 10^{10}$  cm/sec is given in Figure 4.

#### EXPERIMENTAL AND CALCULATED RESULTS

Several inhomogeneous coupled-line filters have been fabricated and tested. The experimental circuits are of the inhomogeneous broadside - coupled strip type shown in Figure 1B and were designed using the impedance and velocity data given by Allen<sup>6</sup>. Figure 5 compares the measured and calculated response of a C-section filter having a 2 to 1 velocity ratio. The test structure was fabricated on a 1" x 2" x 0.025" A1995 substrate. This filter exhibits 3 - peak stopband response and good agreement exists between measured and calculated response. Figure 6 illustrates for the C-section the experimental and theoretical results of substantial deviation from an integral velocity ratio.

#### CONCLUSION

Inhomogeneous coupled-line filters have been designed, analyzed, fabricated and tested. Experimental and theoretical results show such filters to possess desirable novel properties including rippled passband and stopband response from a single filter

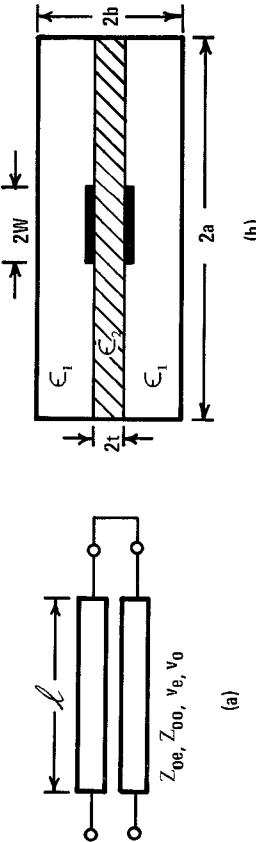


Figure 1. (a) Inhomogeneous C-section. (b) Broadside-coupled strips in a layered dielectric

section.

#### ACKNOWLEDGMENT

The work reported in this paper was jointly sponsored by the National Science Foundation under grant number GK-38117 and by the University of South Florida Research Council.

#### REFERENCES

1. Microwave Filters Using Parallel Coupled Lines, edited by Leo Young, ARTECH House, Inc.
2. Parallel Coupled Lines and Directional Couplers, edited by Leo Young, ARTECH House, Inc.
3. S. B. Cohn, "Parallel Coupled Transmission Line Resonators", Trans. IRE MTT-6 (2), 223-231 (April, 1959).
4. T. G. Bryant and J. A. Weiss, "Parameters of Microstrip Transmission Lines and of Coupled Pairs of Microstrip Lines", Trans. IEEE MTT-16 (2), 1021-1027 (December, 1968).
5. H. E. Brenner, "Use A Computer to Design Suspended Substrate IC's", Microwaves, 38-45 (September 1968).
6. J. L. Allen and M. F. Estes, "Broadside-Coupled Strips in a Layered Dielectric", Trans. IEEE MTT-19(10), 662-669 (October, 1972).
7. E. M. T. Jones and J. T. Bollajahn "Coupled-Strip-Transmission Line Filters and Directional Couplers", Trans. IRE MTT-15 (6), 384-385 (June, 1956).
8. G. I. Zysman and A. K. Johnson, "Coupled Transmission Line Networks in an Inhomogeneous Dielectric Medium", Trans. IEEE MTT-17 (10), 753-759 (October, 1968).

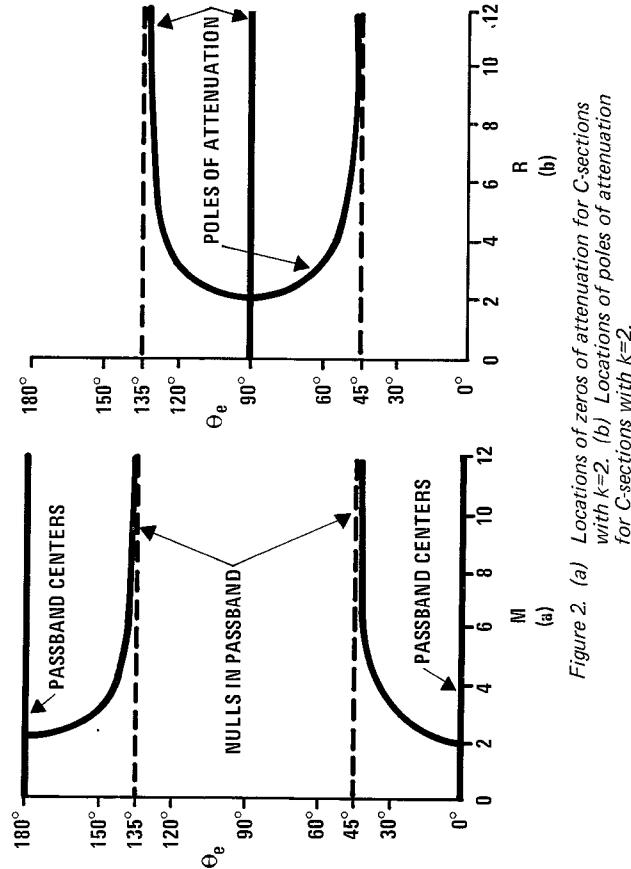


Figure 2. (a) Locations of zeros of attenuation for C-sections with  $k=2$ . (b) Locations of poles of attenuation for C-sections with  $k=2$ .

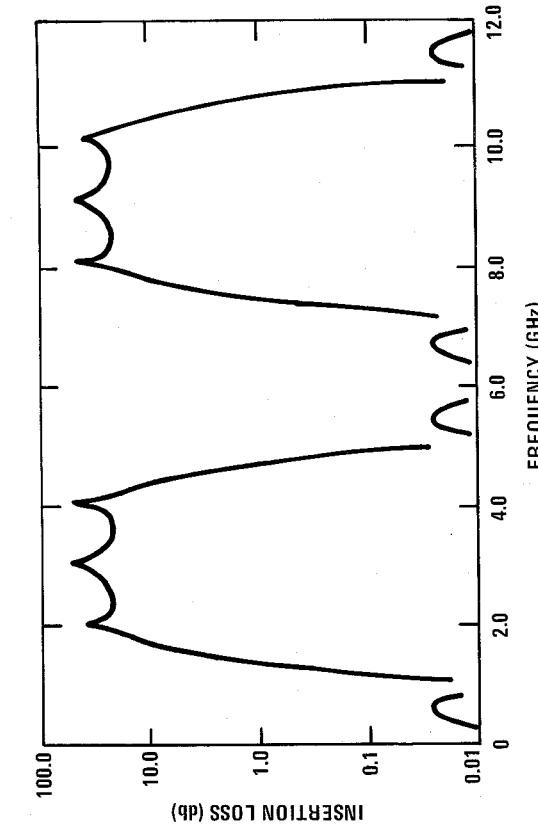


Figure 4. Response of a C-section with  $k=2$ ,  $M=3$ ,  $R=3$ ,  $v_e=2.28 \times 10^{10} \text{ cm/sec}$ ,  $\epsilon_r=1.875 \text{ cm}$ .

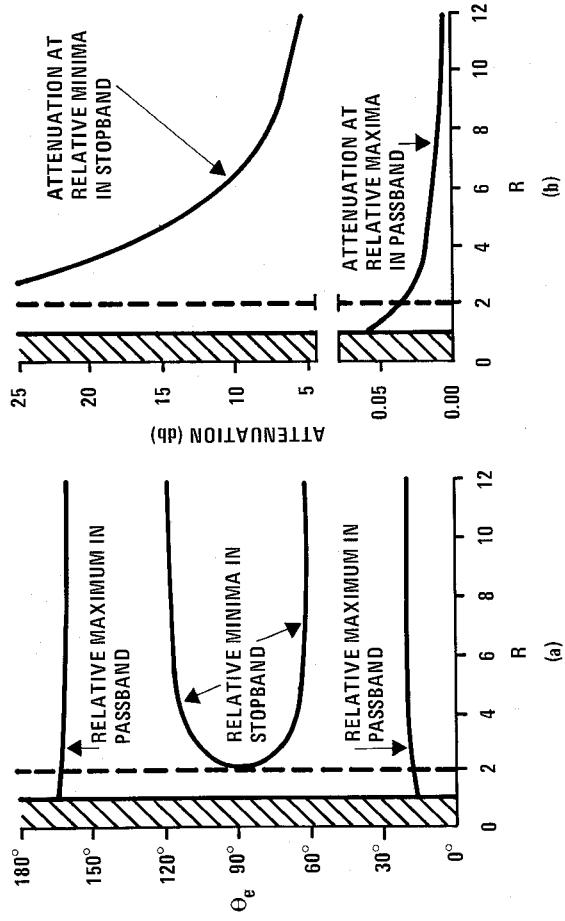


Figure 3. (a) Locations of relative maxima and minima for C-sections with  $k=2$ ,  $M=3$ . (b) Values of attenuation at maxima and minima for C-section with  $k=2$ ,  $M=3$ .

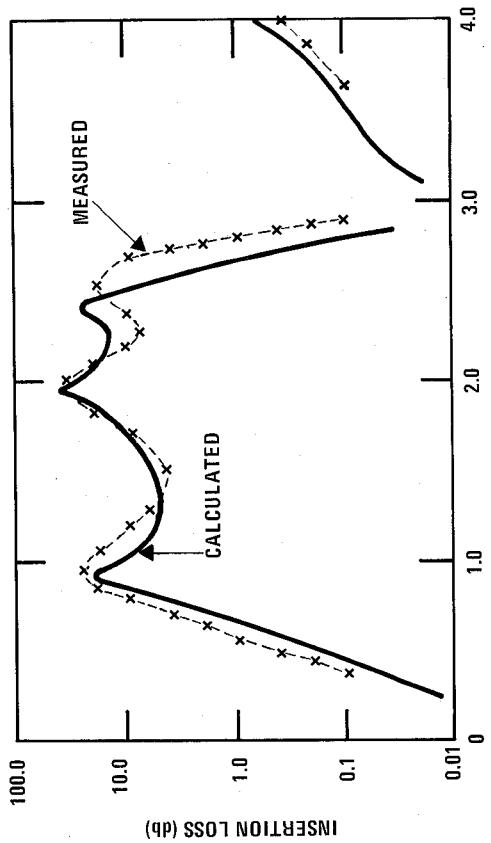


Figure 6. Comparison of Measured and Theoretical Response of a C-section with  $Z_{oe} = 115 \text{ ohm}$ ,  $Z_{oo} = 15 \text{ ohm}$ ,  $v_e = 2.43 \times 10^{10} \text{ cm/sec}$ ,  $\epsilon_r = 3.23 \text{ cm}$ ,  $Z_G = Z_L = 50 \text{ ohm}$ .

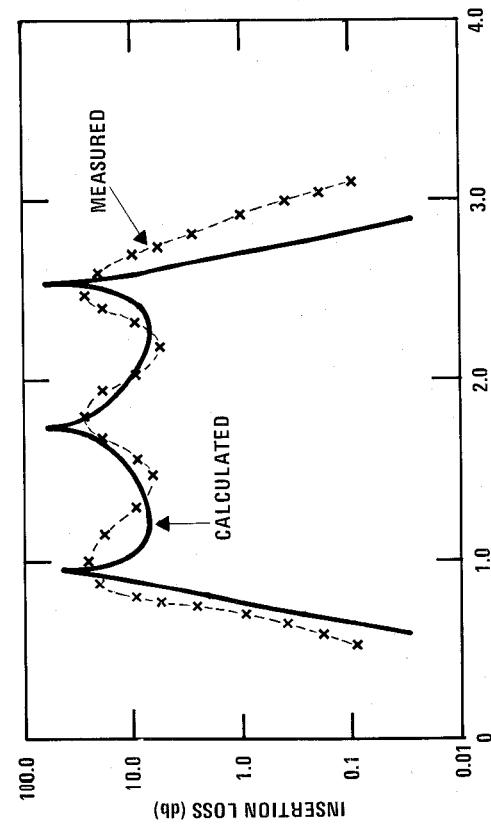


Figure 5. Comparison of Measured and Theoretical Response of a C-section with  $Z_{oe} = 94 \text{ ohm}$ ,  $Z_{oo} = 11 \text{ ohm}$ ,  $v_e = 2.26 \times 10^{10} \text{ cm/sec}$ ,  $\epsilon_r = 1.73 \times 10^{10} \text{ cm/sec}$ ,  $v_0 = 3.23 \text{ cm}$ ,  $Z_G = Z_L = 50 \text{ ohm}$ .